



Advanced Data Structures and Algorithms

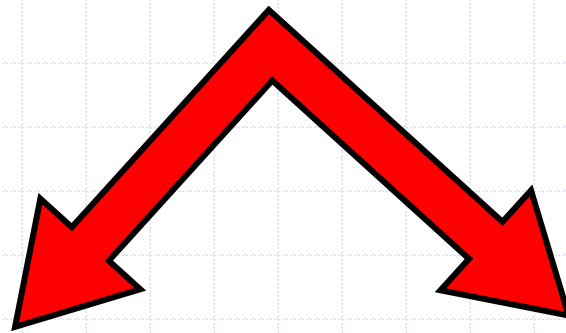
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Department of Computer Science _ UHD

Randomized Data Structures

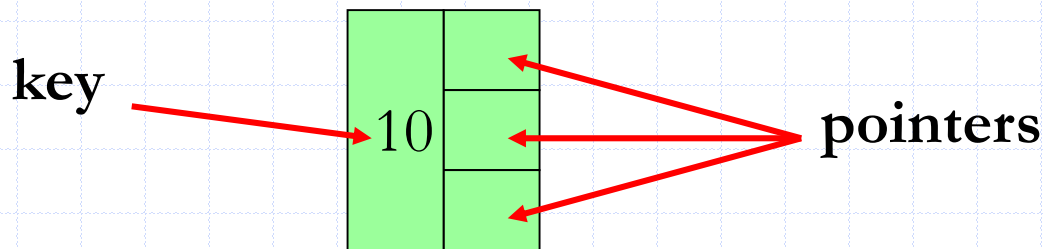


Treaps

Randomized skip lists

What is it?

In computer science, a **skip list** is a data structure that allows **fast search** within an ordered sequence of elements. Fast search is made possible by maintaining a linked hierarchy of subsequences, each skipping over fewer elements.



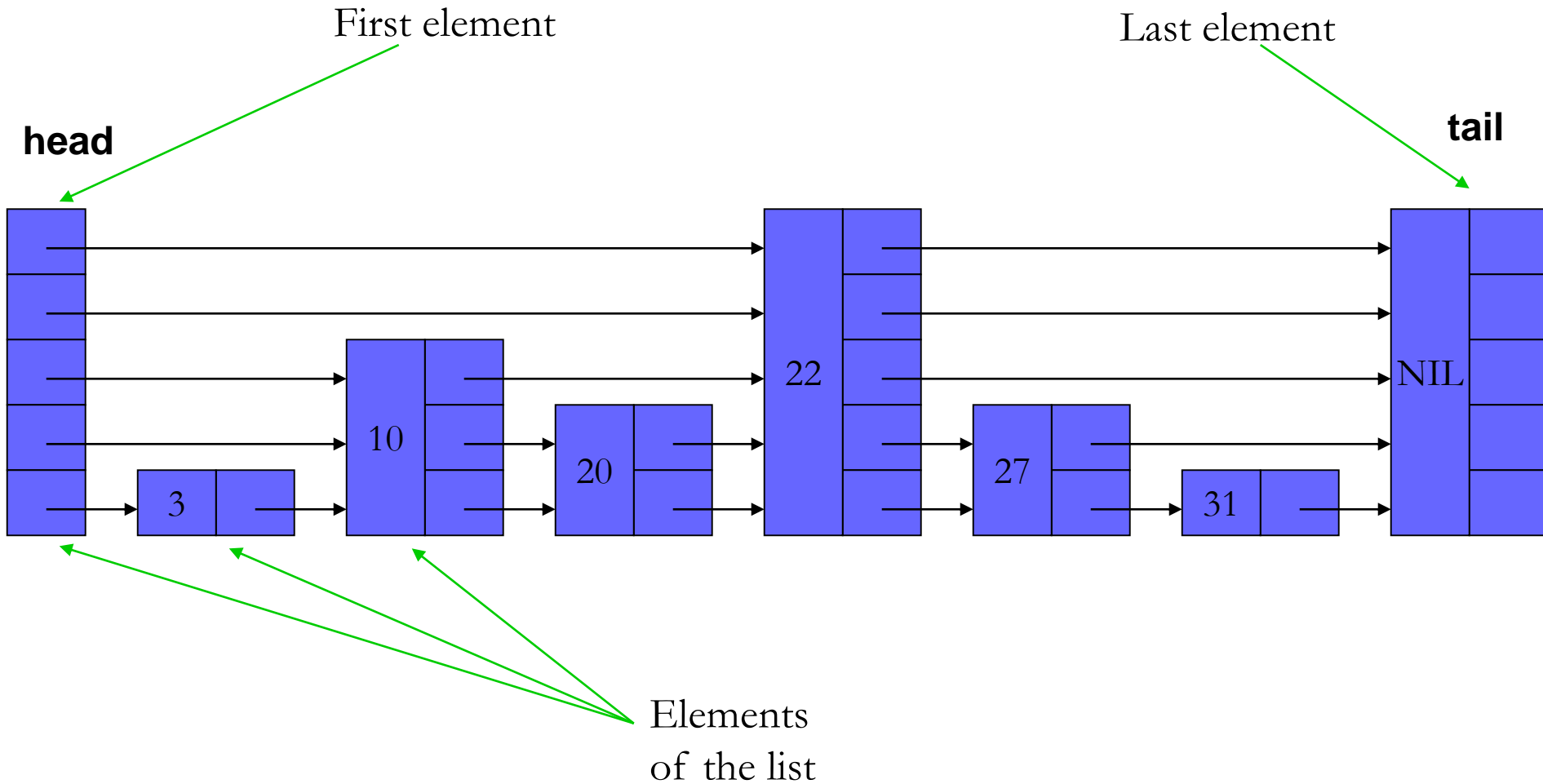
What is it?

- A Skip List is a Data Structure based on parallel linked lists “discovered” by William Pugh in 1990.
- Skip Lists support $O(n)$ time for
 - Insertion
 - Deletion
 - Querying
- At a high level, a skip list is just a **sorted**, singly linked list with some “shortcuts” (additional pointers that allow to **travel faster**).

Skip lists

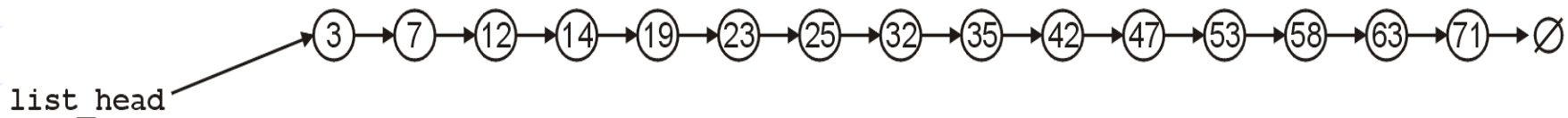
- A linked list has desirable insertion and deletion characteristics if we have a pointer to one of the nodes
 - Searching a linked list with n elements, however is $O(n)$
 - Can we achieve some of the characteristics of an **array** for searching a specific element ? To achieve this in a **linked list**, we require a **pointer** to the central element

What is it?



Skip lists

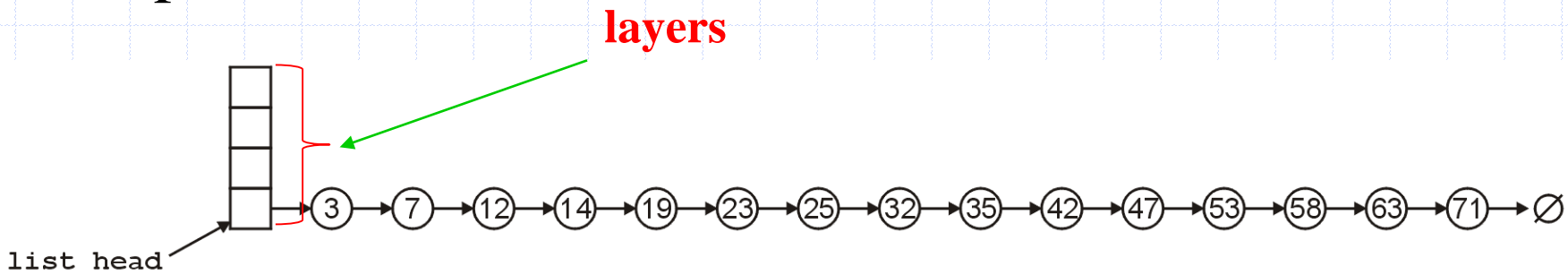
- What if we could skip over many elements at a time?
- Consider the following linked list...



Skip lists

- Start with a sorted linked list
 - Add another **layer** linking every other element
 - Repeat for that layer, etc
- Think of as a hierarchy of sorted linked lists

Suppose, however, if we had an array of head pointers

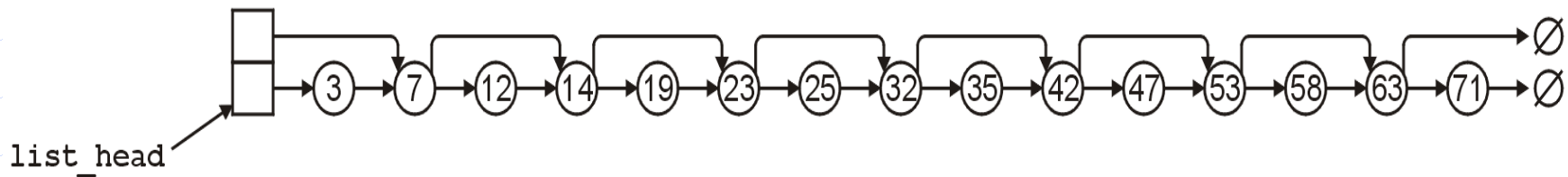


Skip list runtime

- How high does this stack (**layer**) go?
 - Level 1: n
 - Level 2: $n/2$
 - Level 3: $n/4$
- So search through the skip list will be $O(n)$
- But can we maintain this guarantee of efficiency?

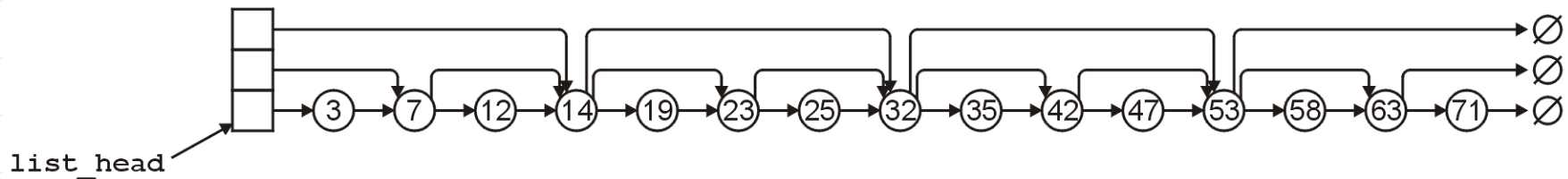
Skip lists

First, we could point to every second node



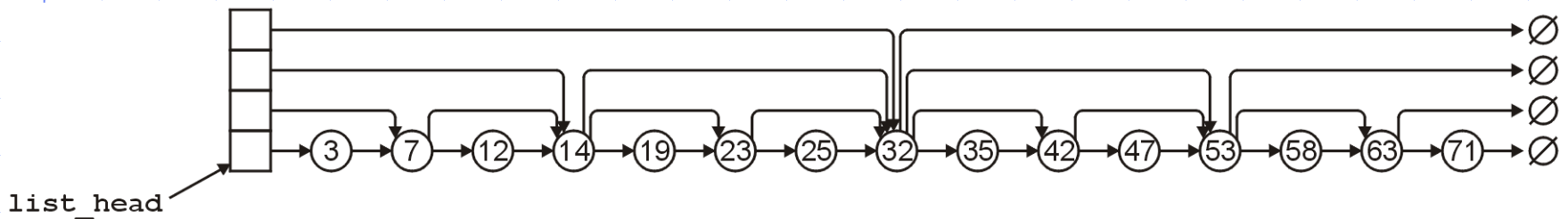
Skip lists

We continue by pointing to every 4th entry



Skip lists

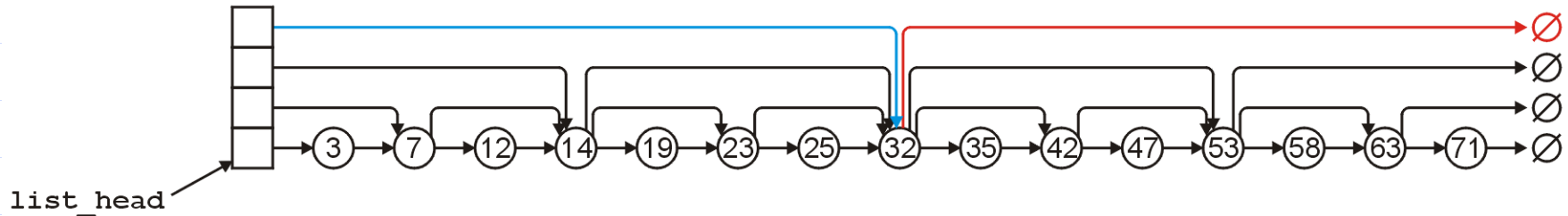
And then every 8th entry, and so on...



Skip lists

Suppose we search for 47

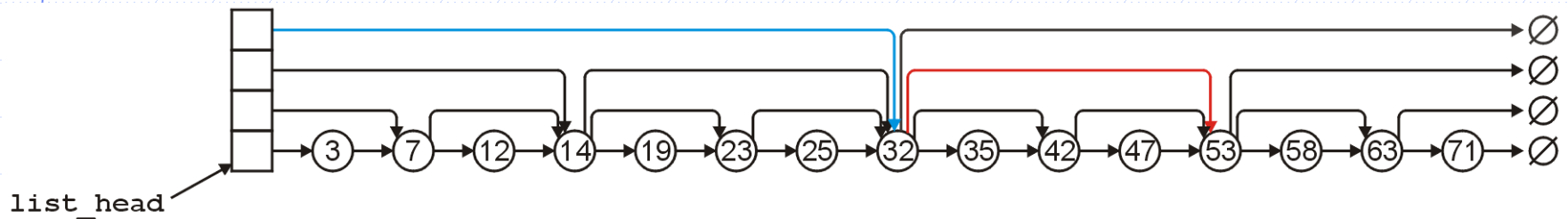
- Following the 4th-level pointer, first $32 < 47$ but the next pointer is 0



Skip lists

Suppose we search for 47

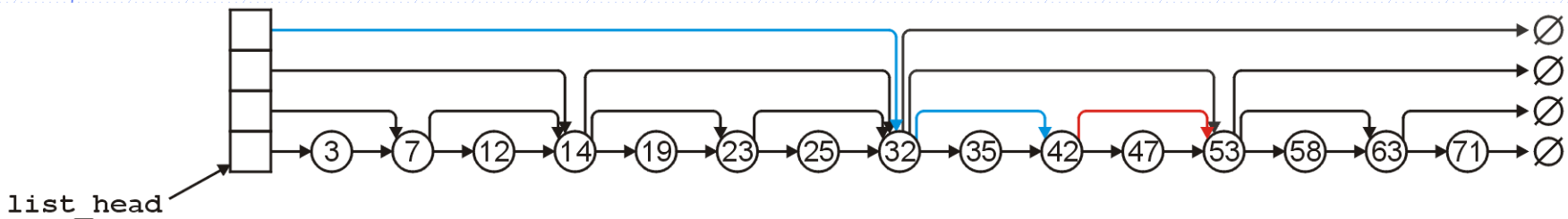
- Continuing with the 3rd-level pointer from 32, $53 > 47$



Skip lists

Suppose we search for 47

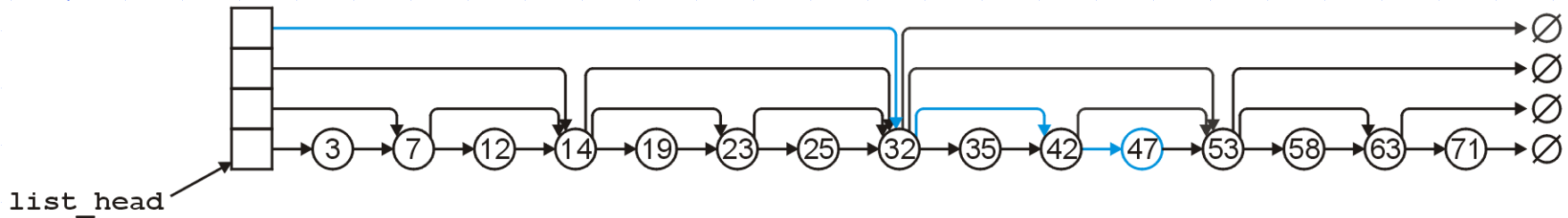
- Continuing with the 2nd-level pointer from 32, $47 > 42$ but $53 < 47$



Skip lists

Suppose we search for 47

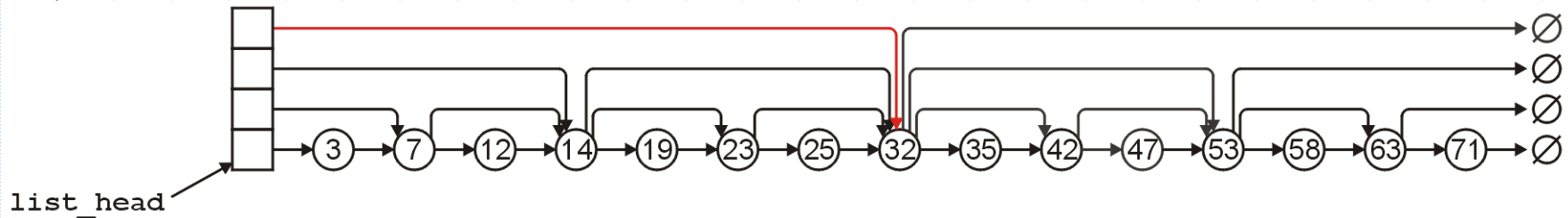
- Continuing with the 1st-level pointer from 42, we find **47**



Skip lists

Suppose we search for 24

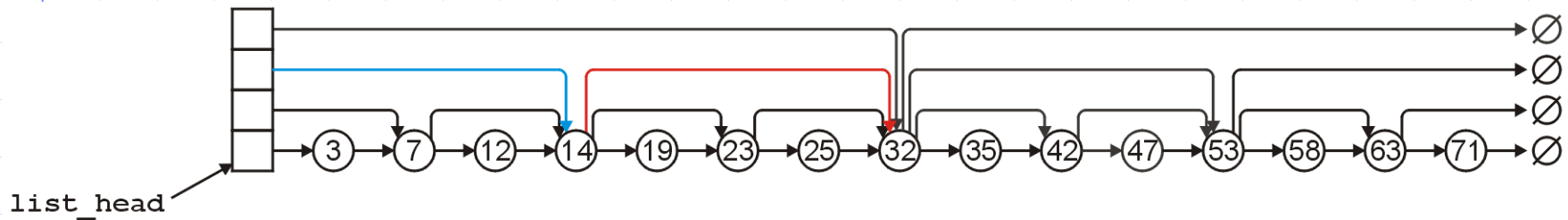
- With the 4th-level pointer, $32 > 24$



Skip lists

Suppose we search for 24

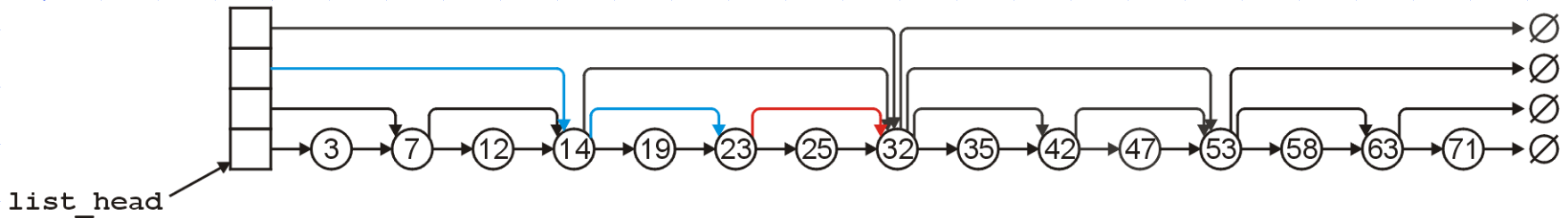
- Following the 3rd-level pointers, $14 < 24$ but $32 > 24$



Skip lists

Suppose we search for 24

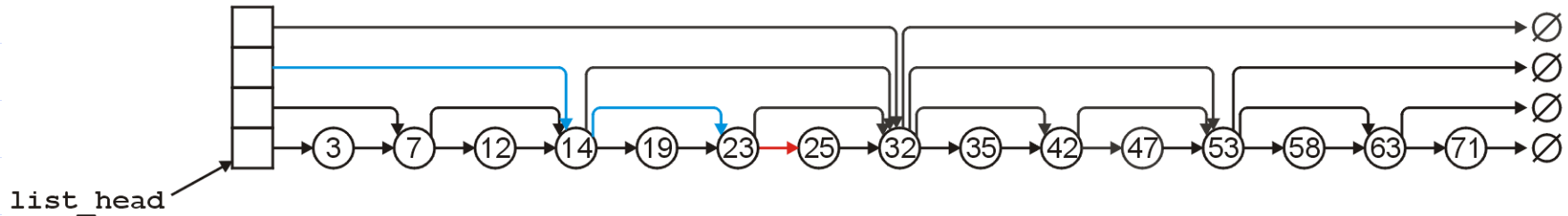
- Following the 2nd-level pointer from 14, $23 < 24$ but $32 > 24$



Skip lists

Suppose we search for 24

- Following the 1st-level pointer from 23, $25 > 24$
- **Thus, 24 is not in the skip list**



SEARCH

Example: search the object which key is 20

Start with the top pointer of the head

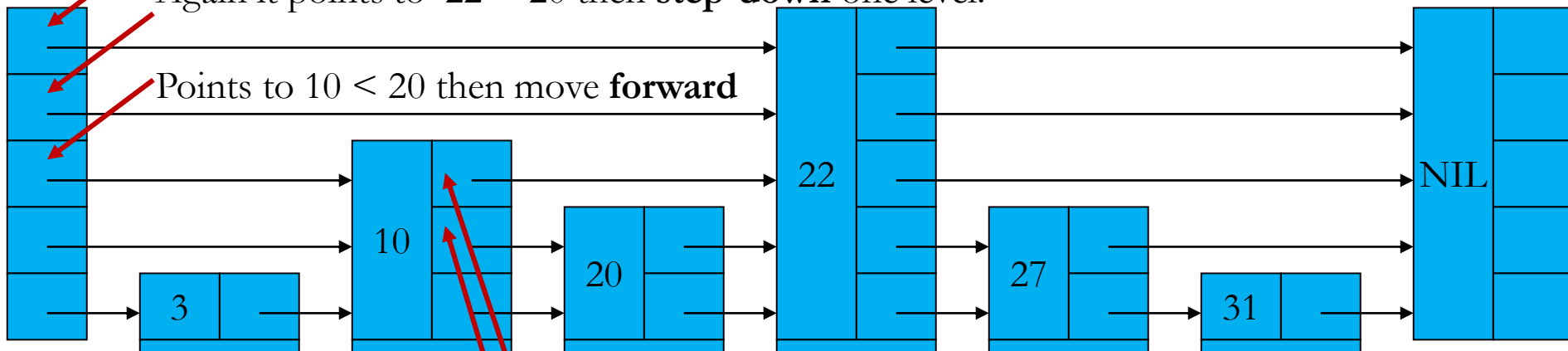
Since it points to 22 > 20 then **step-down** one level.

Again it points to 22 > 20 then **step-down** one level.

Points to 10 < 20 then move **forward**

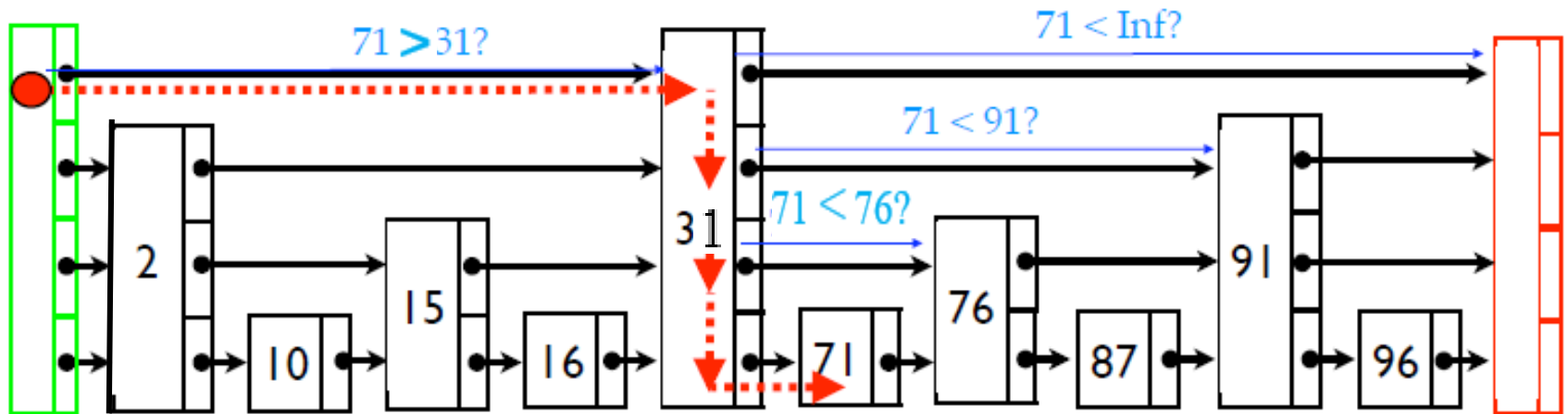
22.> 20 then **step-down**

Element 20 found return



SEARCH

Find 71



When search for k :

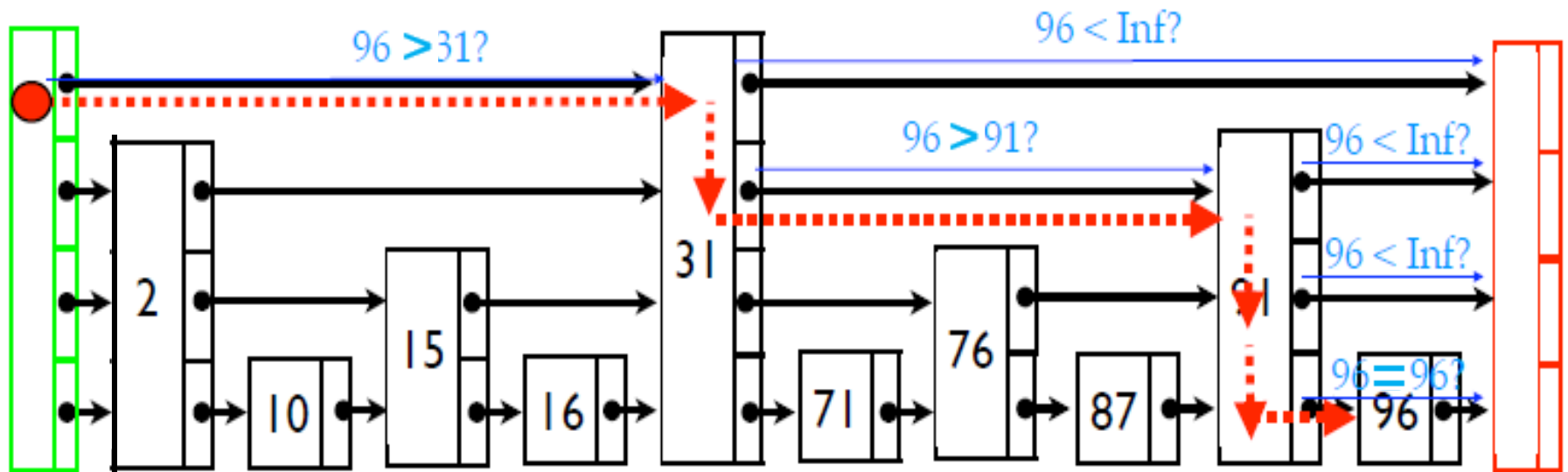
If $k = \text{key}$, done!

If $k < \text{next key}$, go down a level

If $k \geq \text{next key}$, go right

SEARCH

Find 96



When search for k :

If $k = \text{key}$, done!

If $k < \text{next key}$, go down a level

If $k \geq \text{next key}$, go right



Thank you
?????