جامعة التنمية البشرية
UNIVERSITY OF HUMAN DEVELOPMENT

Advanced Data Structures and Algorithms

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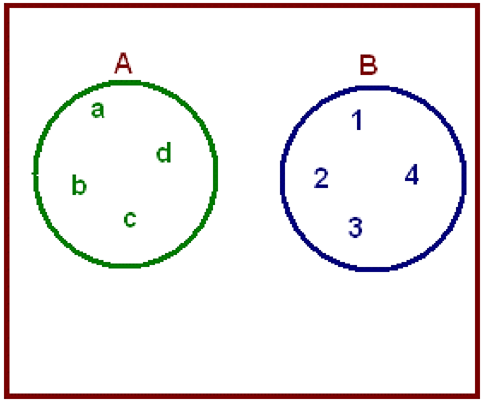
What this Lecture is about:

- A disjoint-set data structure
- Disjoint Set Operations
- An Application of Disjoint-Set
- An Application of Disjoint-Set Data Structures
- Linked-List Implementation
- Linked-lists for two sets
- Disjoint-set Implementation: Forests
- Algorithm for Disjoint-Set
- Example

A disjoint-set data structure

- A disjoint-set is a collection **S**=$\{S_1, S_2, \ldots, S_k\}$ of distinct dynamic sets.
- Is a data structure that keeps track of a set of elements partitioned into a number f disjoint subsets.
- Each set is identified by a member of the set, called *representative*.



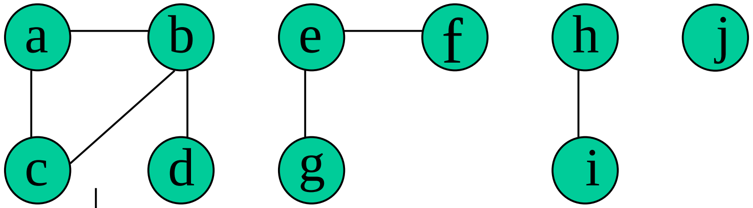
Two disjoint sets A and B

Disjoint set operations:

- MAKE-SET(*x*): create a new set with only *x*. assume *x* is not already in some other set.
- UNION(x,y): combine the two sets containing *x* and *y* into one new set. A new representative is selected.
- FIND-SET(x): returns a pointer to the representative of the unique set containing x. *Find: Determine which subset a particular element is in.*

An Application of Disjoint-Set Data Structures

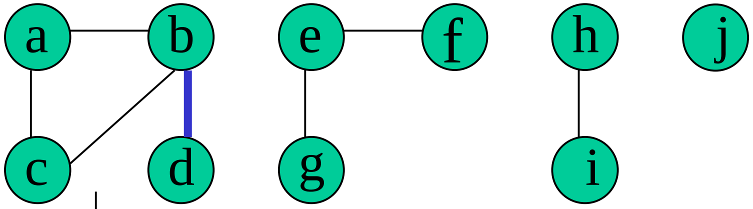
Determining the connected components of an undirected graph G=(V,E)



Initial	{a}	{b}	{c}	{d}	{e}	{f}	{g}	{h}	{i}	{j}

An Application of Disjoint-Set Data Structures

Determining the connected components of an undirected graph G=(V,E)



Initial	{a}	{b}	{c}	{d}	{e}	{f}	{g}	{h}	{i}	{j}
(b, d)	{a}	{b, d}	{c}		{e}	{f}	{g}	{h}	{i}	{j}

An Application of Disjoint-Set Data Structures

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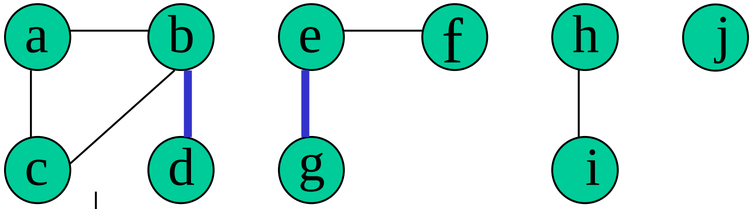

Initial	{a}	{b}	{c}	{d}	{e}	{f}	{g}	{h}	{i}	{j}
(b, d)	{a}	{b, d}	{c}		{e}	{f}	{g}	{h}	{i}	{j}
(e, g)	{a}	{b, d}	{c}		{e, g}	{f}		{h}	{i}	{j}

An Application of Disjoint-Set Data Structures

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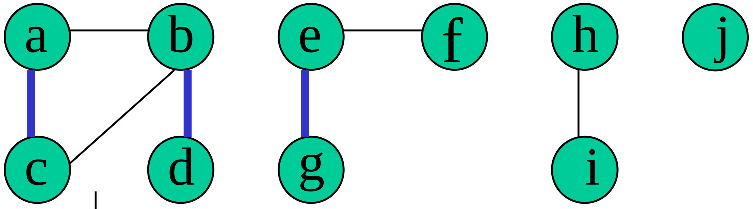

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(a, c)	{a, c}	{b, d}			{e, g}	{f}		{h}	{i}	{j}

An Application of Disjoint-Set Data Structures

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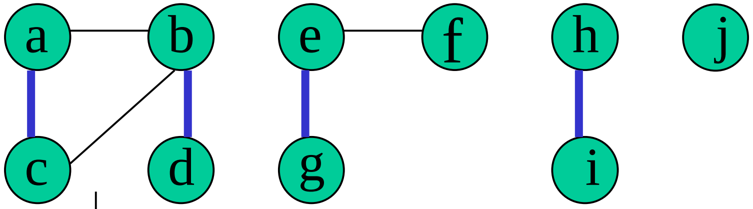

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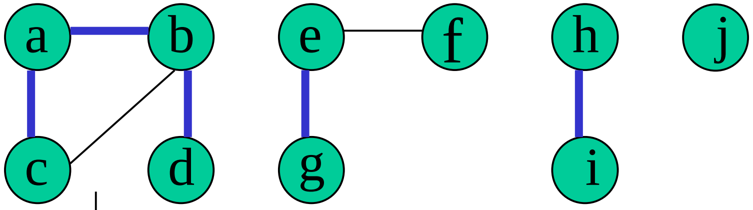
(e, g)	{a}	{b, d}	{c}		{e, g}	{f}		{h}	{i}	{j}
(a, c)	{a, c}	{b, d}			{e, g}	{f}		{h}	{i}	{j}
(h, i)	{a, c}	{b, d}			{e, g}	{f}		{h, i}		{j}

An Application of Disjoint-Set Data Structures

Determining the connected components of an undirected graph G=(V,E)



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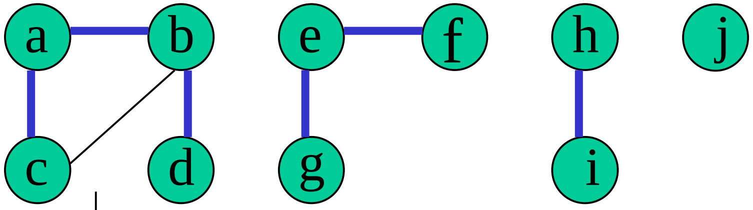
(b, d)	{a}	{b, d}	{c}		{e}	{f}	{g}	{h}	{i}	{j}
(e, g)	{a}	{b, d}	{c}		{e, g}	{f}		{h}	{i}	{j}
(a, c)	{a, c}	{b, d}			{e, g}	{f}		{h}	{i}	{j}
(h, i)	{a, c}	{b, d}			{e, g}	{f}		{h, i}		{j}
(a, b)	{a, b, c, d}				{e, g}	{f}		{h, i}		{j}

An Application of Disjoint-Set Data Structures

Determining the connected components of an undirected graph G=(V,E)



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An Application of Disjoint-Set Data Structures

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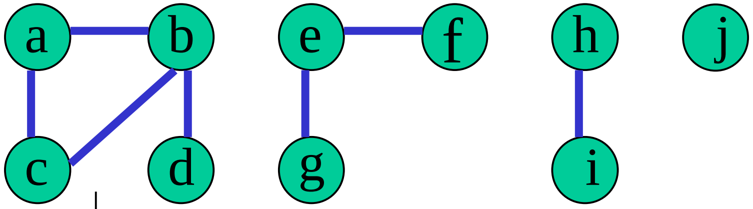

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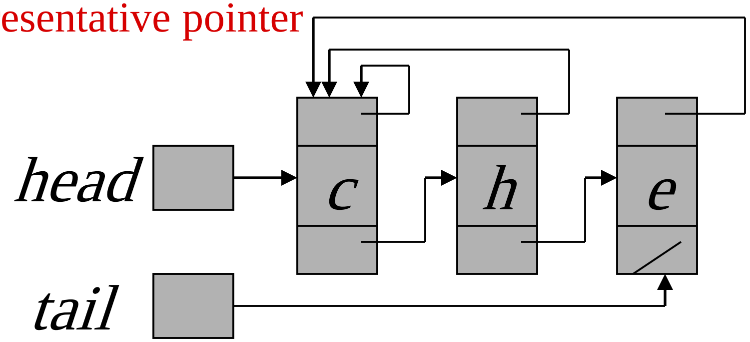
(b, c)	{a, b, c, d}				{e, f, g}			{h, i}		{j}

Linked-List Implementation

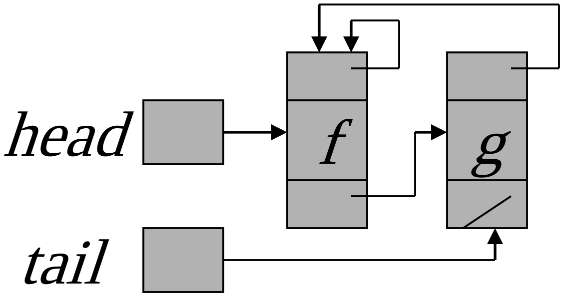
- Each set as a linked-list, with head and tail, and each node contains value, next node pointer and back-to-representative pointer.
- *Example:*
- MAKE-SET costs $O(1)$: just create a single element list.
- FIND-SET costs $O(1)$: just return back-to-representative pointer.

Linked-lists for two sets

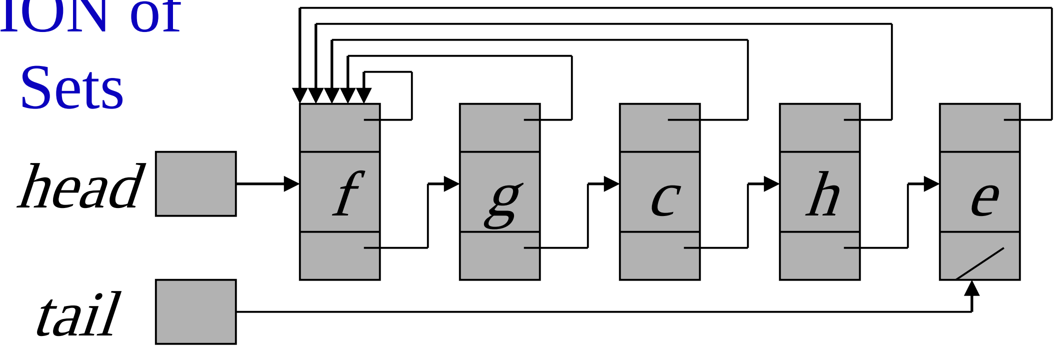
Set {*c*,*h*,*e*}


Representative pointer

Set {*f*, *g*}

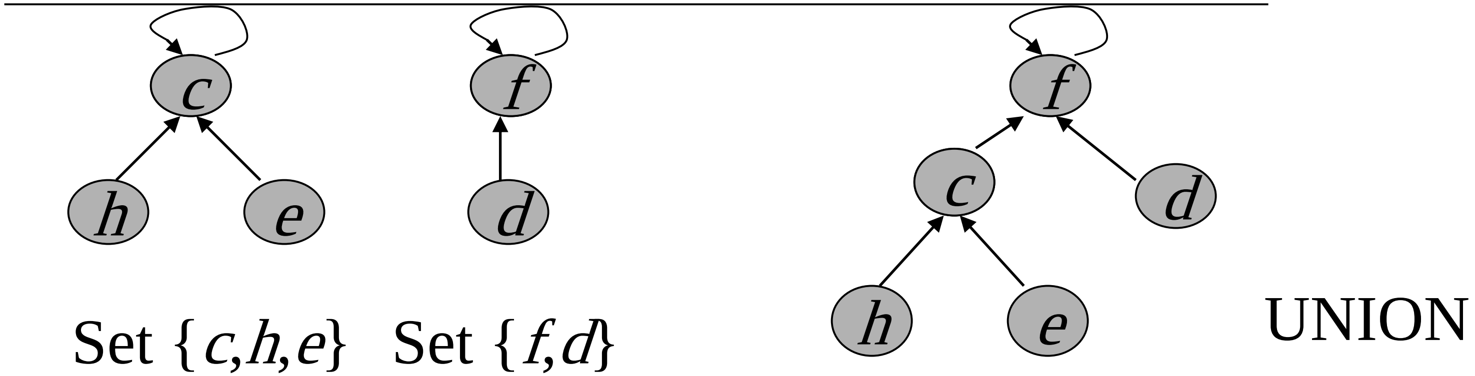


UNION of two Sets



Disjoint-set Implementation: Forests

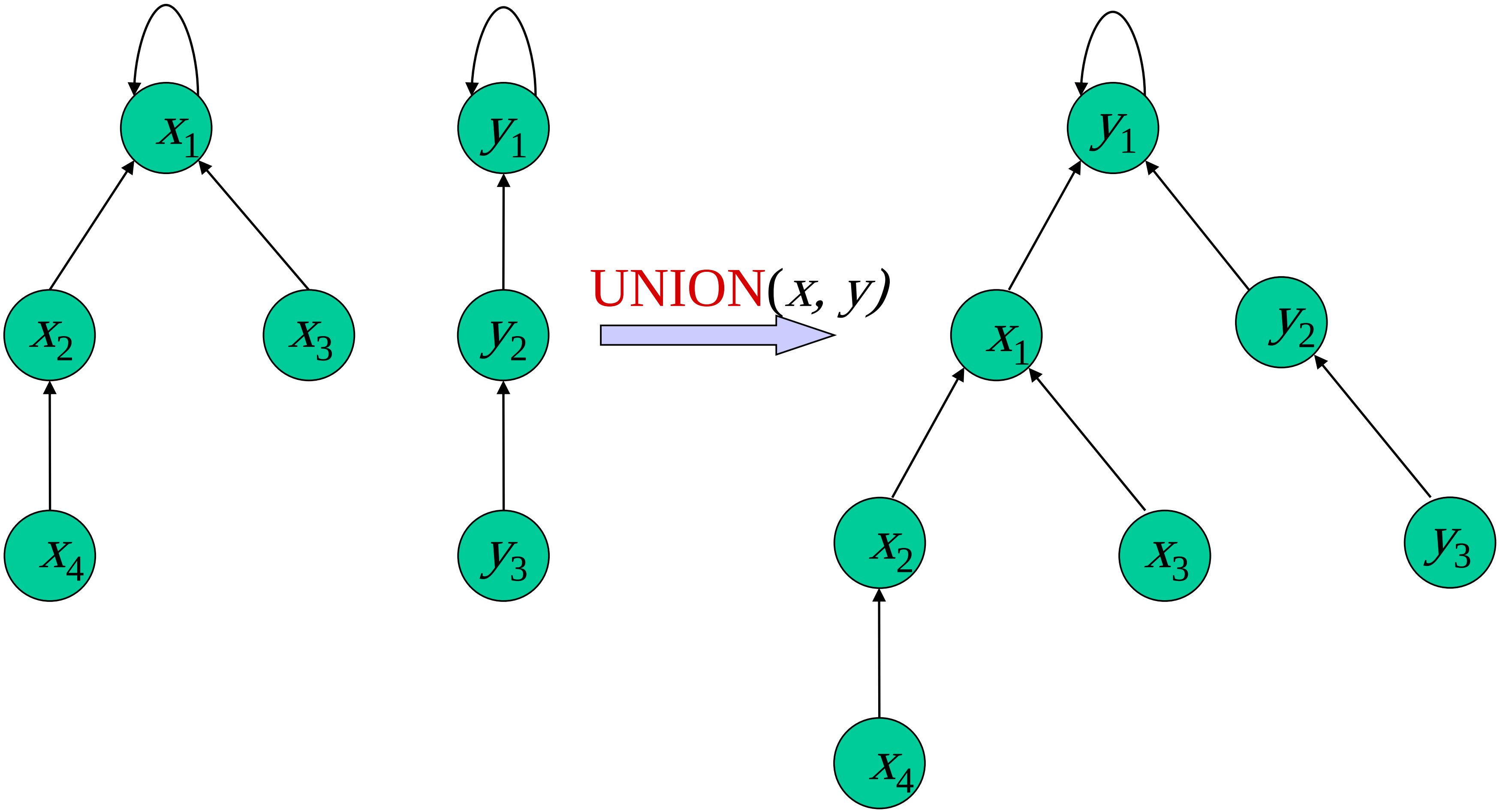
- Rooted trees, each tree is a set, root is the **representative**. Each node points to its parent. Root points to itself.



Set $\{c,h,e\}$ Set $\{f,d\}$

UNION

Disjoint Set Forests

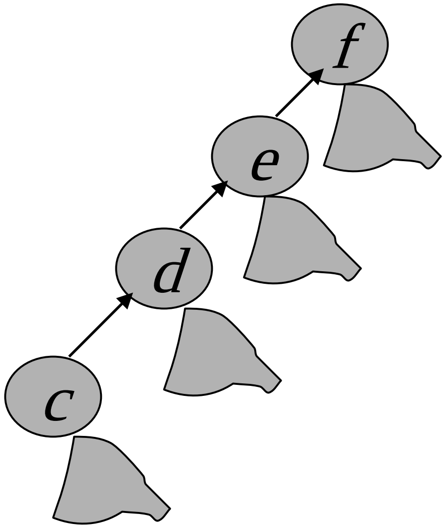


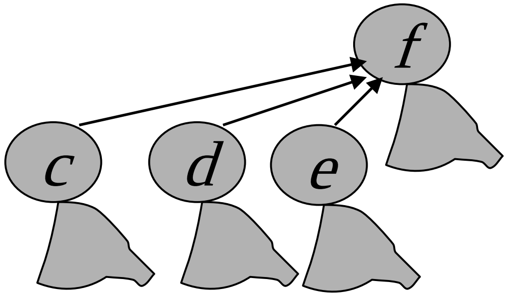
Disjoint Set Forests

Path Compression

Path Compression: used in FIND-SET(*x*) operation, make each node in the path from *x* to the root directly point to the root. Thus reduce the tree height.

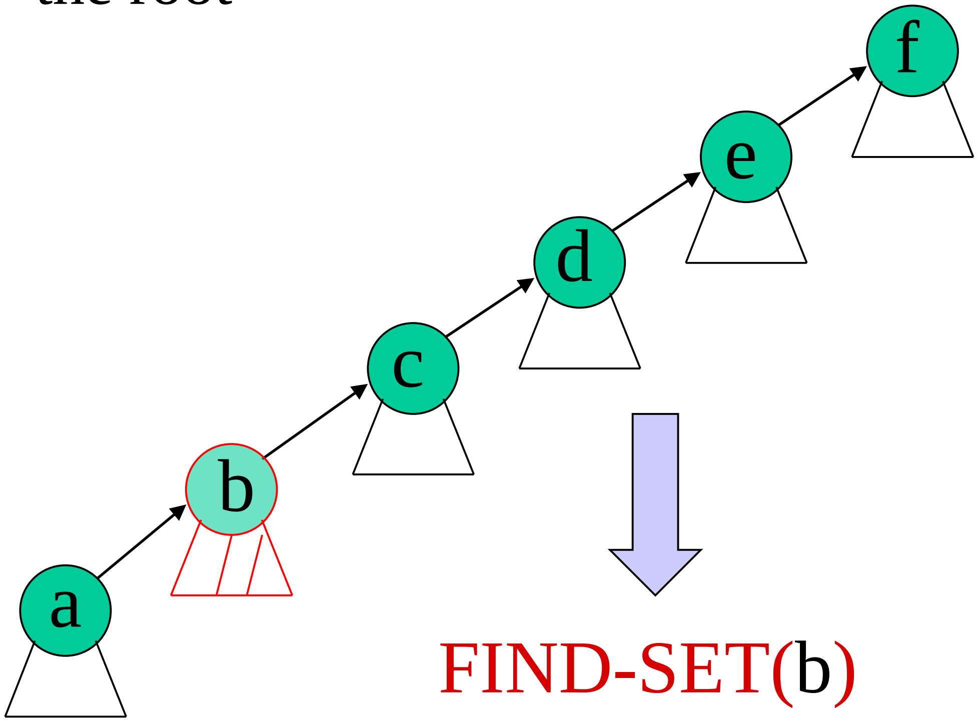
Path Compression





Path Compression

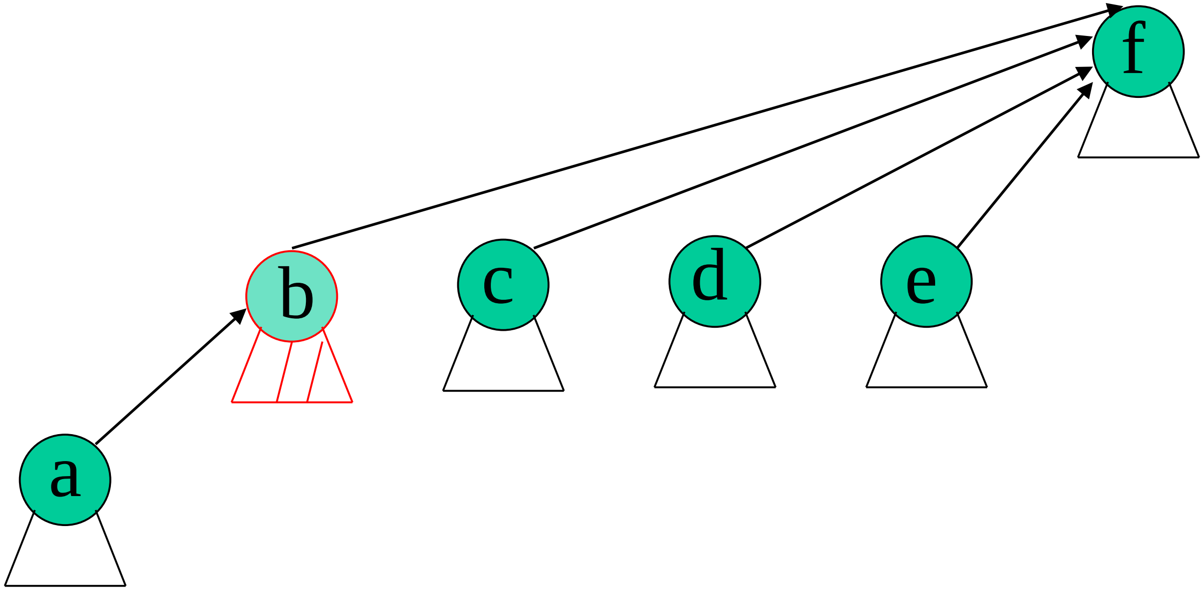
- Use it during the FIND-SET operations
- Make each node on the FIND-PATH to point directly to the root



FIND-SET(b)

Path Compression

Path Compression During FIND-SET(b) Operation



Algorithm for Disjoint-Set

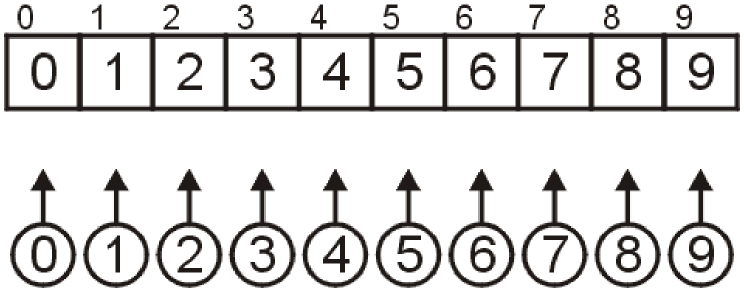
```
function MakeSet(x)
    x.parent := x
```

```
function Find(x)
    if x.parent == x
        return x
    Else
        return Find(x.parent)
```

```
function Union(x, y)
    xRoot := Find(x)
    yRoot := Find(y)
    xRoot.parent := yRoot
```

Example

Consider the following disjoint set on the ten decimal digits:



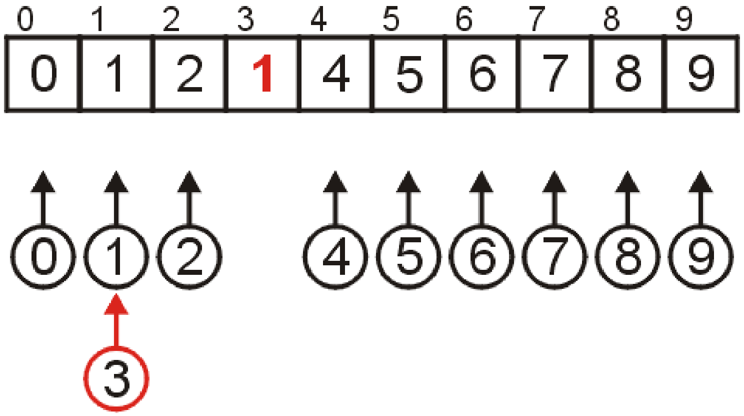
{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}

Example

If we take the union of the sets containing 1 and 3

```
set_union(1, 3);
```

we perform a find on both entries and update the second

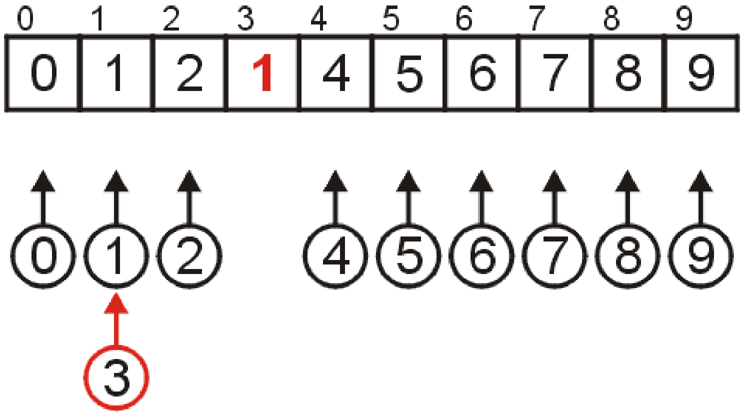


0	1	2	3	4	5	6	7	8	9
0	1	2	1	4	5	6	7	8	9

{0}, {1, 3}, {2}, {4}, {5}, {6}, {7}, {8}, {9}

Example

Now, **`find(1)`** and **`find(3)`** will both return the integer 1



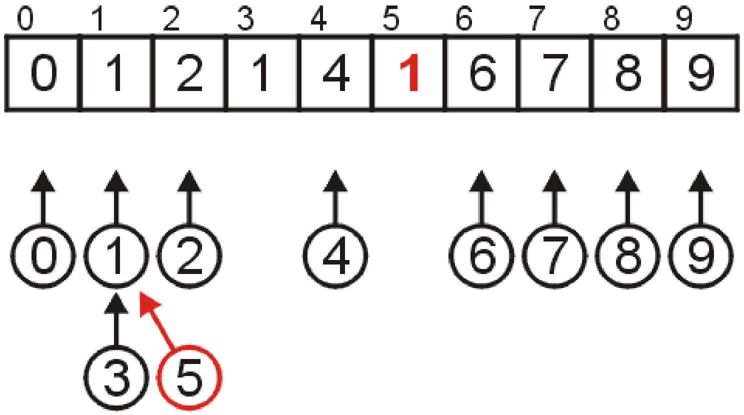
{0}, {1, 3}, {2}, {4}, {5}, {6}, {7}, {8}, {9}

Example

Next, take the union of the sets containing 3 and 5,

`set_union(3, 5);`

we perform a find on both entries and update the second



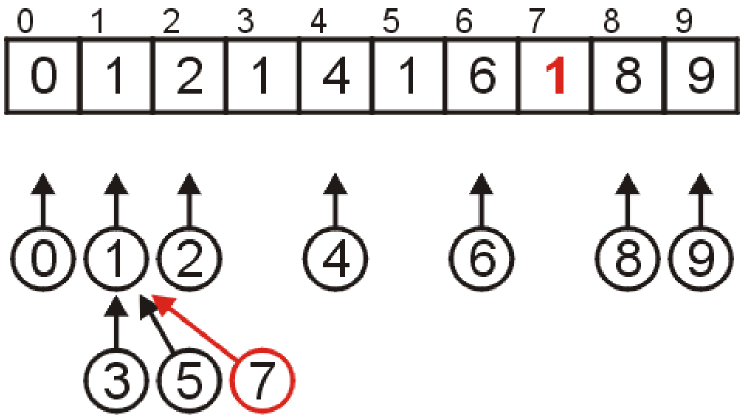
{0}, {1, 3, 5}, {2}, {4}, {6}, {7}, {8}, {9}

Example

Now, if we take the union of the sets containing 5 and 7

```
set_union(5, 7);
```

we update the value stored in `find(7)` with the value `find(5)`:



0	1	2	3	4	5	6	7	8	9
0	1	2	1	4	1	6	1	8	9

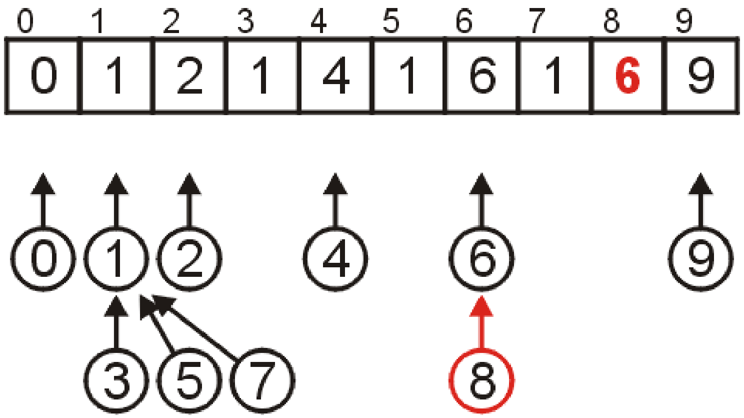
$\{0\}, \{1, 3, 5, 7\}, \{2\}, \{4\}, \{6\}, \{8\}, \{9\}$

Example

Taking the union of the sets containing 6 and 8

`set_union(6, 8);`

we update the value stored in `find(8)` with the value `find(6)`:



0	1	2	3	4	5	6	7	8	9
0	1	2	1	4	1	6	1	**6**	9

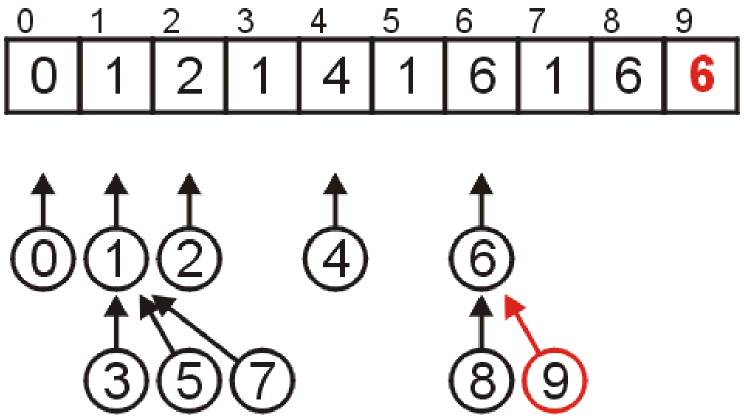
{0}, {1, 3, 5, 7}, {2}, {4}, {6, 8}, {9}

Example

Taking the union of the sets containing 8 and 9

`set_union(8, 9);`

we update the value stored in `find(8)` with the value `find(9)`:



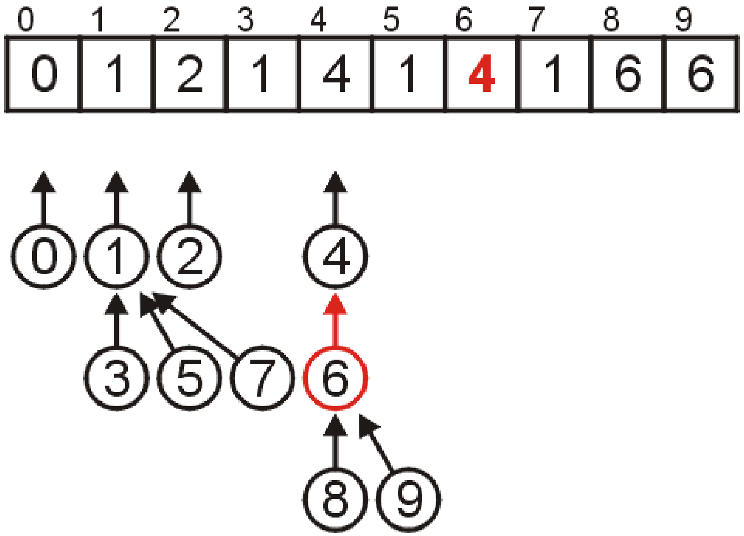
{0}, {1, 3, 5, 7}, {2}, {4}, {6, 8, 9}

Example

Taking the union of the sets containing 4 and 8

`set_union(4, 8);`

we update the value stored in `find(8)` with the value `find(4)`:



0	1	2	3	4	5	6	7	8	9
0	1	2	1	4	1	**4**	1	6	6

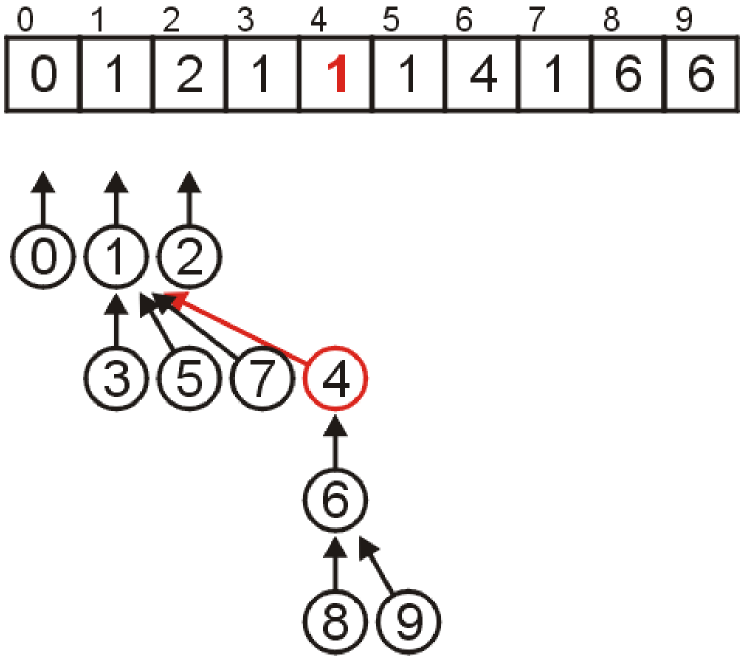
{0}, {1, 3, 5, 7}, {2}, {4, 6, 8, 9}

Example

Finally, if we take the union of the sets containing 5 and 6

```
set_union(5, 6);
```

we update the entry of `find(6)` with the value of `find(5)`:



0	1	2	3	4	5	6	7	8	9
0	1	2	1	1	1	4	1	6	6

{0}, {1, 3, 4, 5, 6, 7, 8, 9}, {2}

THANK YOU

???