Advanced Data Structures and Algorithms

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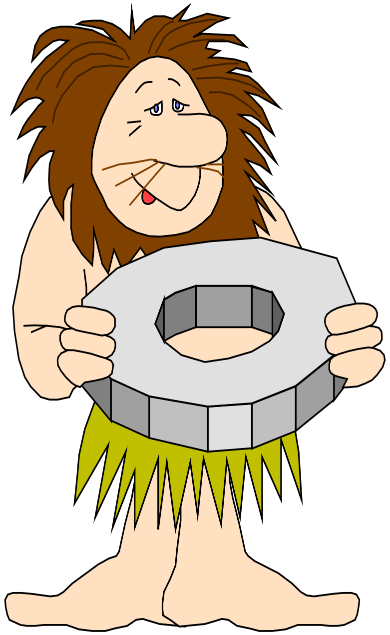
What this Lecture is about:

- Petri Net as a Graph
- Petri Net Components
- Example of Petri Net
- Petri Net – terminology
- Firing a Transition
- Petri Net - Transition enabling
- Enabling condition
- Initial marking



Petri Net as a Graph

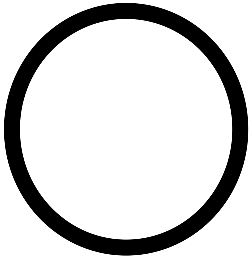
Petri net was developed in the early 1960s by Carl Adam Petri's dissertation from Germany.

A **Petri net** is a directed graph, in which the nodes represent transitions (i.e. events that may occur, signified by bars) and places (i.e. conditions, signified by circles).

Petri Net as a Graph

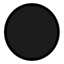
- Graph contains 2 types of nodes
  - Circles (Places)
  - Bars (Transitions)
- Petri net has dynamic properties that result from its execution
  - Markers (Tokens)
  - Tokens are moved by the firing of transitions of the net.

Petri Net Components

Circles (Places)  **Pi**

Bars (Transitions)  **Ti**

Arcs 

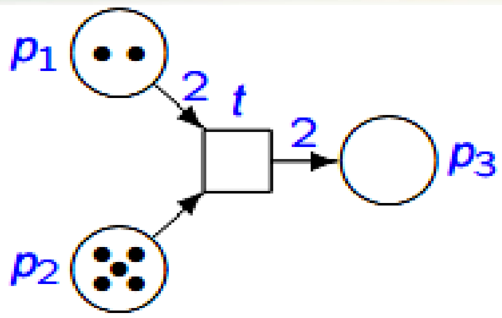
Marker (Tokens) 

Place/Transition Net

A Place/Transition Net (P/T net) is a tuple $N = \langle P, T, F, W, M_0 \rangle$, where

- P is a finite set of places,
- T is a finite set of transitions,
- the places P and transitions T are disjoint ($P \cap T = \emptyset$),
- $F \subseteq (P \times T) \cup (T \times P)$ is the flow relation,
- $W\colon F \to (\mathbb{N} \setminus \{0\})$ is the arc weight mapping, and
- $M_0 : P \to \mathbb{N}$ is the initial marking representing the initial distribution of tokens.

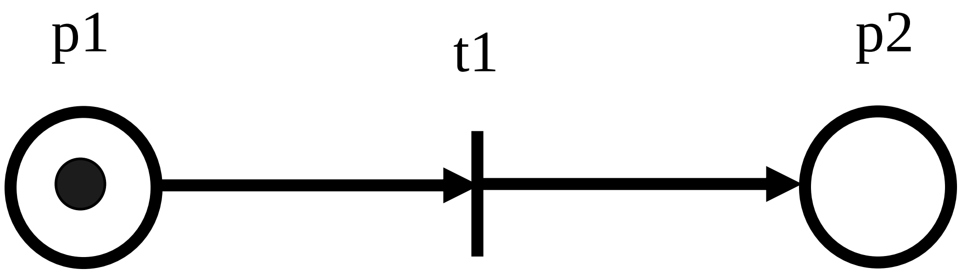
Place/Transition Net: Example



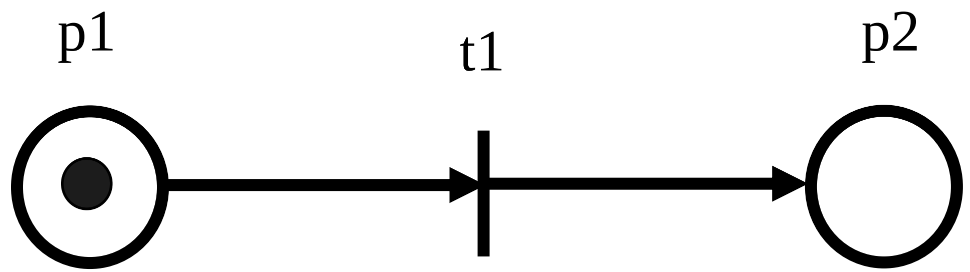
The place/transition net $\langle P, T, F, W, M_0 \rangle$ above is defined as follows:

- $P = \{p_1, p_2, p_3\}$,
- $T = \{t\}$,
- $F = \{\langle p_1, t\rangle, \langle p_2, t\rangle, \langle t, p_3\rangle\}$,
- $W = \{\langle p_1, t\rangle \mapsto 2, \langle p_2, t\rangle \mapsto 1, \langle t, p_3\rangle \mapsto 2\}$,
- $M_0 = \{p_1 \mapsto 2, p_2 \mapsto 5, p_3 \mapsto 0\}$.

Example of Petri Net



Petri Net - terminology



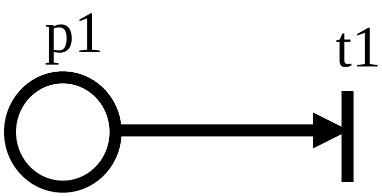
This Petri net has:

- 2 places: p1, p2
- 1 transition: t1
- p1 has 1 token: f(p1) = 1
- p2 has 0 tokens: f(p2) = 0

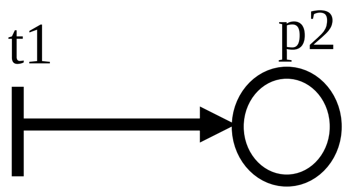
Firing a Transition

When a transition **tj** fires

- Each **pi** that has an edge from **pi** to **tj** removes a token from **pi**

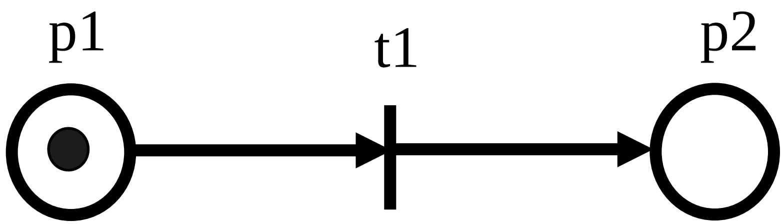


- Each **pj** that has an edge from **tj** to **pj** adds a token to **pj**

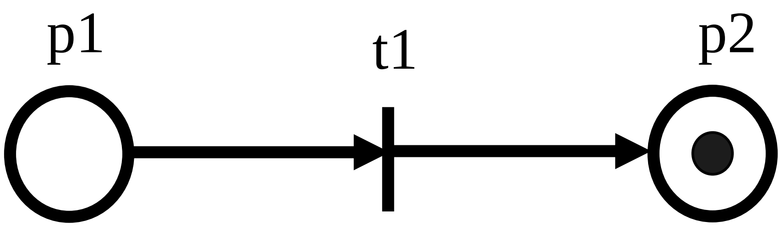


Firing a Transition

Petri net before t1 fires:



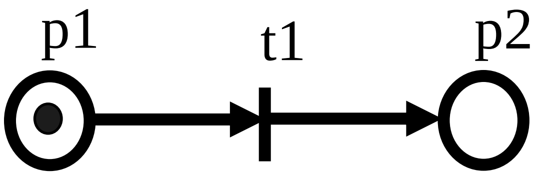
Petri net after t1 fires:



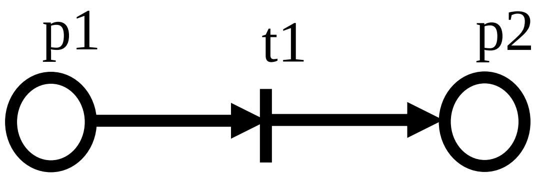
Firing a Transition

A transition must be enabled before it fires:

There is a token in each input place pi that has an edge from place to the transition.



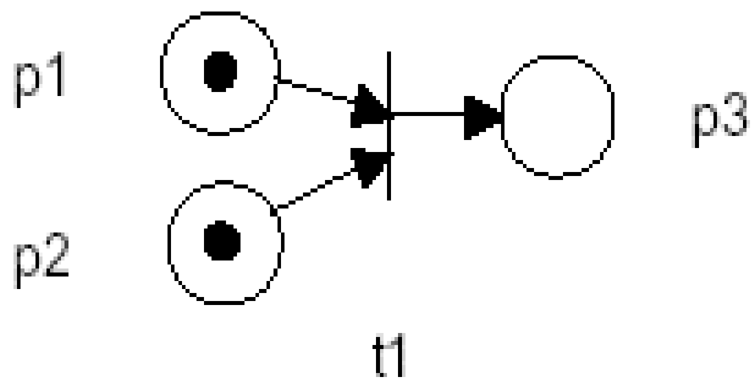
the transition t1 can fire



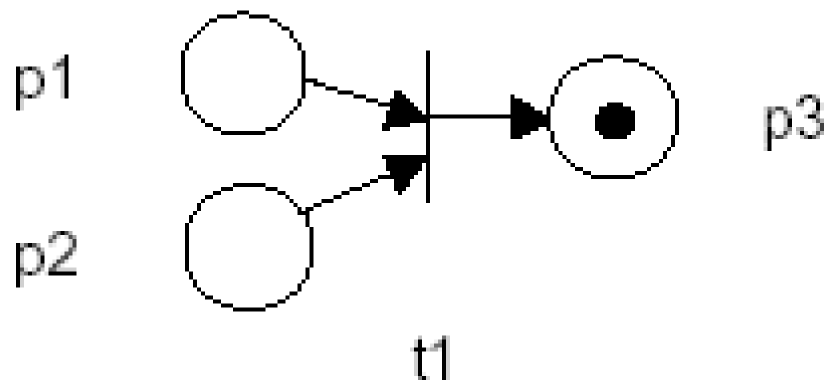
the transition t1 cannot fire

Firing a Transition

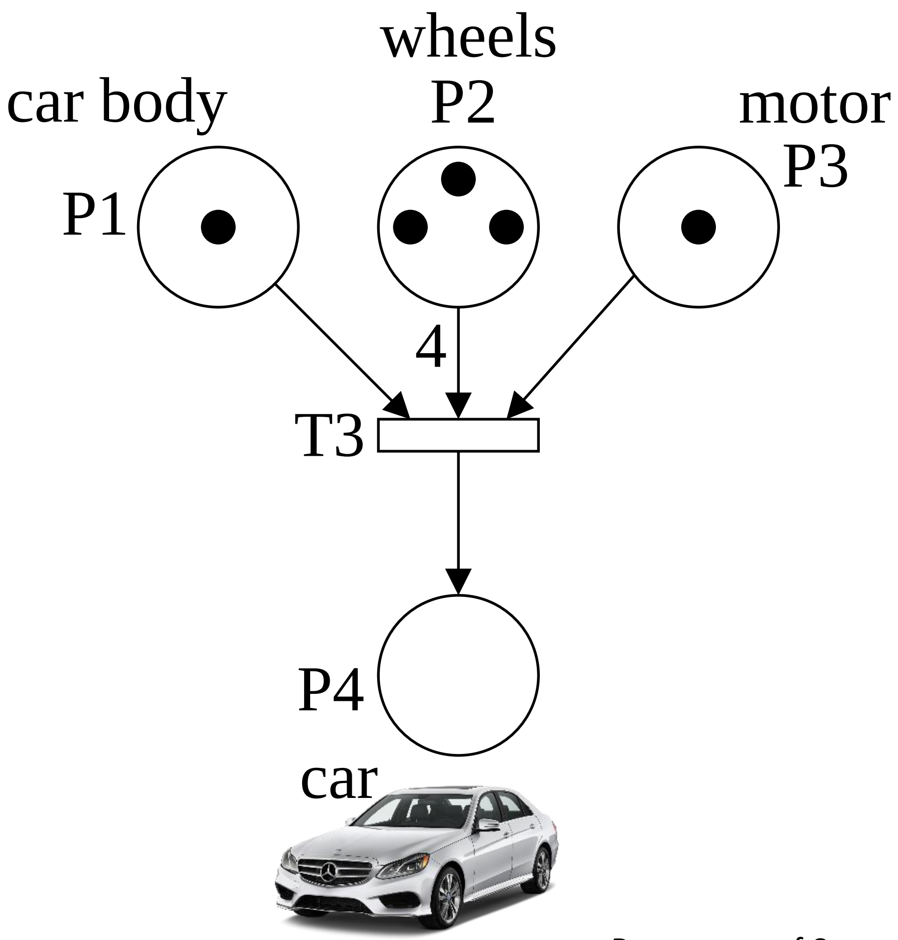
Petri net before t1 fires:



Petri net after t1 fires:



Petri Net - Transition enabling - example



- weight of the P2 - T3 edge is 4
- place P2 contains only 3 3 tokens
- transition T3 is not enabled (for one car we need four wheels)
- transition T3 cannot be fired

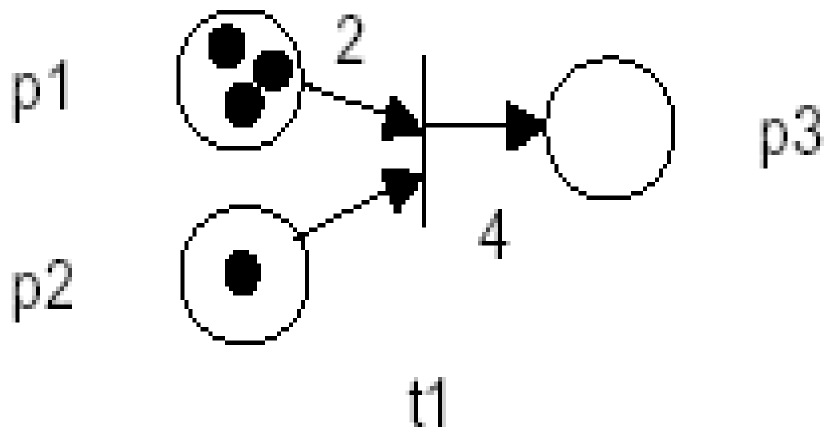
Original Petri Nets

Weighted Petri Nets

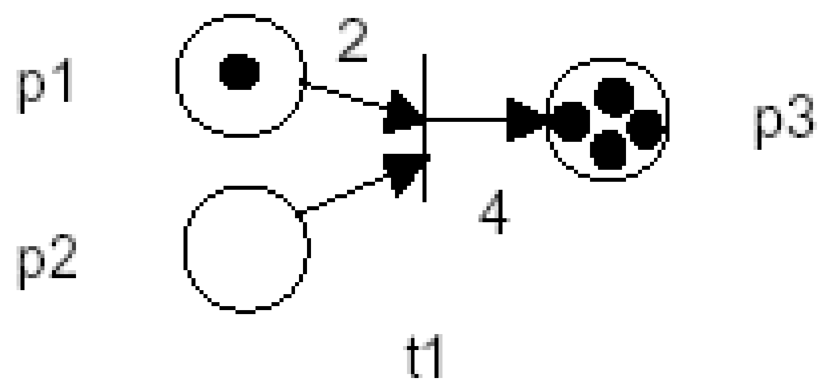
- Generalized the original Petri net to allow *multiple tokens* to be added/removed when a transition fires.
- The edges are labeled with the weight (i.e., number of tokens)
- If there is no label, then the default value is 1

Original Petri Nets

Weighted Petri net before t1 fires:

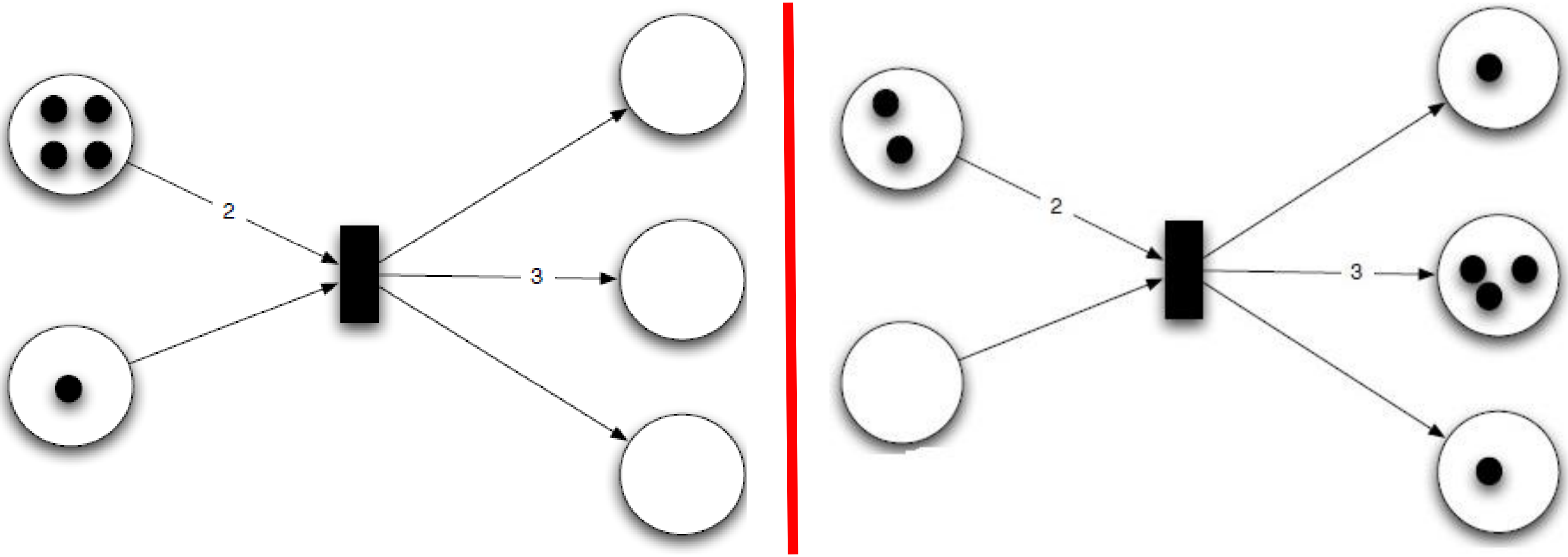


Weighted Petri net after t1 fires:



Firing a transition

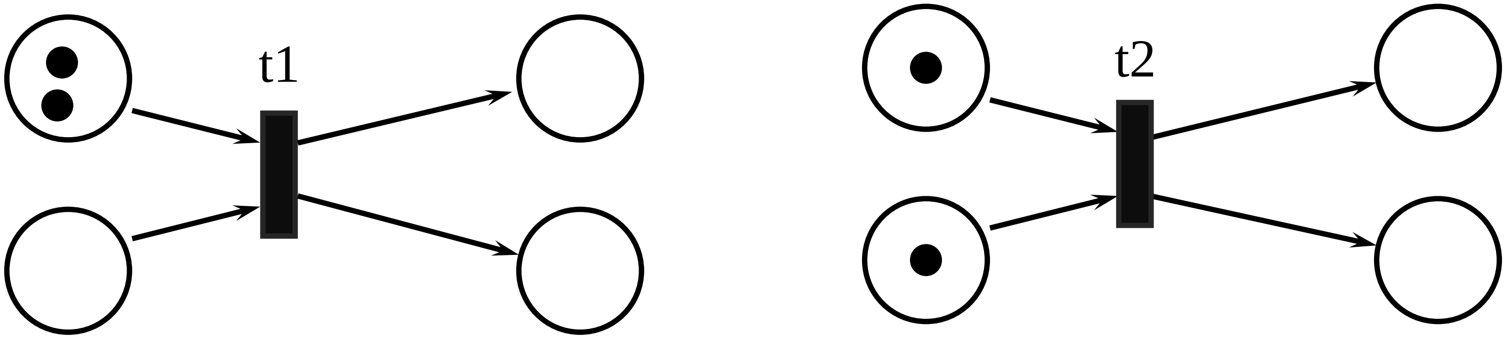
Transitions consume tokens from the input places and produce tokens in the output places



Enabling condition

Transitions are the active components and places and tokens are **passive**.

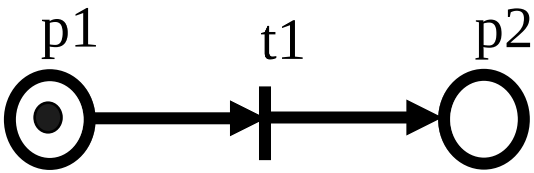
A transition is **enabled** if each of the input places contains tokens.



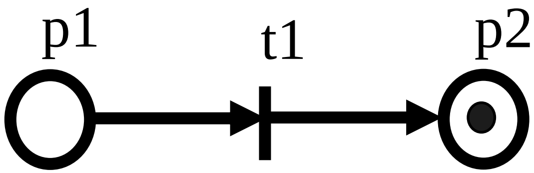
Transition t1 is not enabled, transition t2 is enabled.

Initial marking

Petri net before t1 fires:



Petri net after t1 fires:



Initial marking M0 = {1 , 0}

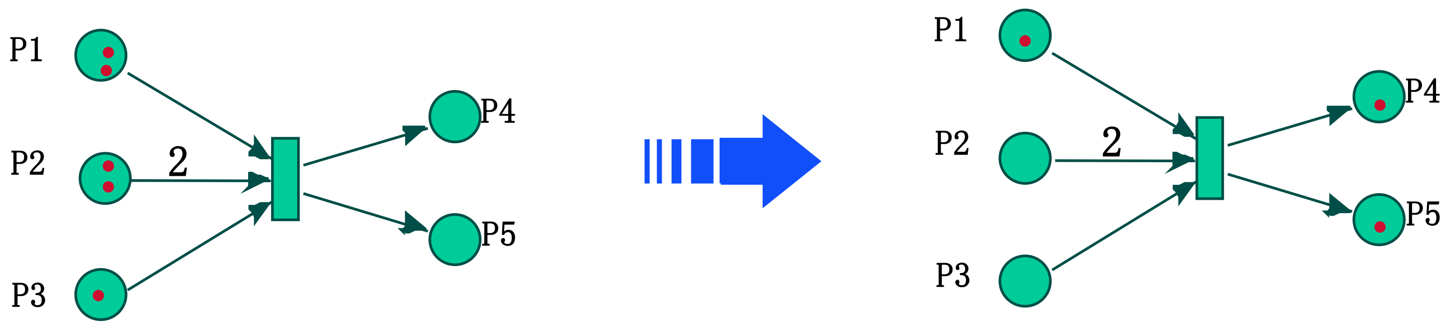
New marking M1 = {0 , 1}

Initial marking

Initial marking M0 = {1 , 1 , 0}

New marking M1 = {0 , 0 , 1}

Initial marking



Initial marking M0 = {2 , 2 , 1 , 0 , 0}

New marking M1 = {1 , 0 , 0 , 1 , 1}

Petri net example

$P = \{p_1, p_2, p_3, p_4\};$

$T = \{t_1, t_2, t_3\};$

$I(t_1, p_1) = 2,$

$I(t_2, p_2) = 1,$

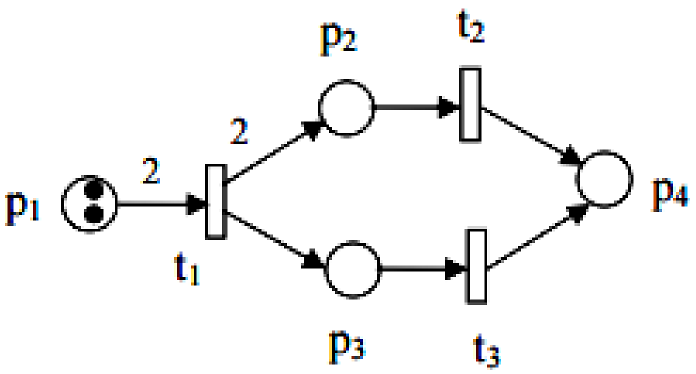
$I(t_3, p_3) = 1,$

$O(t_1, p_2) = 2, O(t_1, p_3) = 1,$

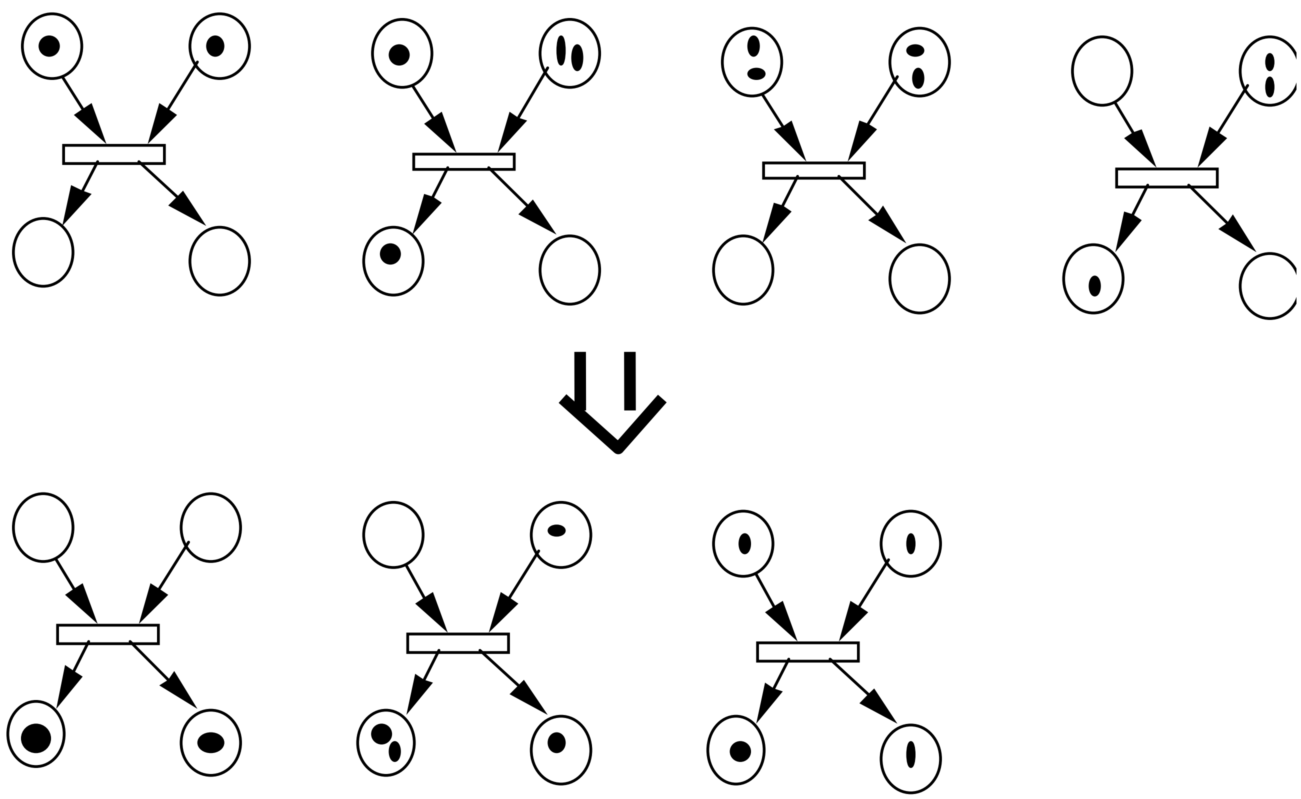
$O(t_2, p_4) = 1,$

$O(t_3, p_4) = 1,$

$M_0 = (2\ 0\ 0\ 0)^T.$



Example



Thank you